ABSTRACT

The concept of fuzzy sets was introduced by Prof. L. A. Zadeh in his classical paper [103]. After the discovery of the fuzzy subsets, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a modern theory of fuzzy topology is developed. The notion of fuzzy subsets naturally plays a significant role in the study of fuzzy topology which was introduced by C. L. Chang [21] in 1968. In 1980, Pu and Liu [83, 84] introduced the concepts of quasi-coincidence and Q-neighbourhoods and the notion of a fuzzy point (some times called a fuzzy singletons) which again opened up a new avenue by providing with a new fuzzy methodology by which extensions of different aspects in fuzzy setting can very interestingly and effectively be carried out.

The concept of operations (γ) on a topological space was introduced by S. Kasahara [53] for the first time in 1979. The author unified several known characterization of compact spaces, nearly compact spaces and H-closed spaces by introducing a certain operation on a topology. Then his works drew attention of many Topologists viz. Jankovic, Ogata, Umehara, Maki, Noiri, F. U. Rehaman, B. Ahmad, S. Husian,S. Tahiliani and so on. D.S Jankovic [46] defined the concept of operation-closure and investigated some properties of functions with operation-closed graphs. By using an operation γ on topological space (X, T), H. Ogata [79] investigated the notion of operation-open sets, i.e. γ-open sets and used it to investigate some new separation axioms. In 1992, F. U. Rehman and B. Ahmad [89] defined and investigated several

properties of γ-interior, γ-exterior, γ-closure and γ-boundary points in topological spaces and studied the characterizations of (γ, β)-continuous mappings initiated by H. Ogata [88]. After that H. Maki, T. Nori, J. Umehara, H. Ogata [71, 80, 95] generalized the notion of operation-open sets to bioperations and defined bioperation-closures and bioperation-generalized closed sets. In addition to these, they obtained some bioperation-continuities, bioperation-separation axioms and bioperation-normal spaces. Rencently B. Ahmad and S.Hussian did many works in terms of γ-operations. They derived different topological concepts and discussed their several properties [7, 8, 9, 10, 11, 12, 13].

Ever since the introduction of Fuzzy Set by Zadeh and Fuzzy Topological Space by Chang, several authors tried to successfully generalize numerous pivot concepts of general topology to fuzzy setting. Till now no attempt has been made to generalize the concept of γ-operations to fuzzy framework. We intended to work in that direction and some theories and results are contributed to the fuzzy literature in the present thesis. Throughout the thesis we try to apply the methodology as initiated by Ming and Ming [84], where the concepts of Q-neighbourhood, quasi-coincidence and fuzzy point play a predominant role.

The first chapter of the thesis gives the introduction and background of different kinds of work done by various authors, which are related directly to our definitions and results. Most of concepts, notations and definitions which we have used throughout the thesis are standard by now. But for the sake of completeness we have enclosed the second chapter as Prerequisites, which incorporates the basic definitions and results of fuzzy sets which are ready references for our work in the subsequent chapters. Results here are mentioned without proof and can be seen in the papers referred to.

In chapter 3, we introduce and study the concept of γ-operation on a fuzzy topological space (X,T). Then we defined notions of fuzzy γ-open (γ-closed) sets and study its related topics like fuzzy γ-interior (fuzzy γ-closure), fuzzy γ-continuous mapping and fuzzy γ-compact in the light of the notions of q-neighbourhoods, quasi-coincidence and fuzzy points. Also we defined fuzzy γ-convergence and fuzzy γ-accumulation of fuzzy filter base and make use of each of them to characterize fuzzy γ-compact. Moreover, we investigate more topological properties of these notions.

In chapter 4, we introduce the concept of fuzzy γ-closed graphs and study in connection with fuzzy γ-continuity, fuzzy γ-subcontinuity, fuzzy γ-compactness and fuzzy filterbase. Then we study the notions of fuzzy locally γ-closed functions and particularly, the notions of fuzzy locally closed function, fuzzy locally θ-closed function and fuzzy locally δ-closed function. Then we developed the concept of fuzzy γ-closed mapping (fuzzy almost γ-closed) which leads to the generalization of the fuzzy closed function (res. Almost-closed), fuzzy θ-closed mapping (res. fuzzy almost θ-closed) and fuzzy δ-closed mapping and (res. fuzzy almost δ-closed). Inspired by the above work on γ-operations, an attempts has also been made to investigate some new fuzzy γ-separation axioms.

In fifth chapter, we defined fuzzy (γ,β)-continuous mappings which unify several characterization and properties of fuzzy continuity, fuzzy θ-continuity, fuzzy δ-continuity, fuzzy weak-continuity, fuzzy strong θ-continuity, fuzzy super continuity and fuzzy weak θ-continuity. Then we furnish the notions of fuzzy (γ,β)-open and fuzzy (γ,β)-closed mappings. Moreover we investigate the concept of fuzzy (γ,β)-

homeomorphism, and particularly, fuzzy homeomorphism, fuzzy θ-homeomorphism and fuzzy δ-homeomorphism. Several characterizations and properties of these mappings are also studied.

In the sixth and last chapter, we have generalized the notion of fuzzy operation-open sets to fuzzy bioperations and introduce the concepts of fuzzy (γ, γ')-open sets for two operations γ and γ' on fuzzy topology T. By using the sets we define and investigate some fuzzy bioperation-separation axioms. Next we introduce an alternative fuzzy bioperation-open set $[\gamma, \gamma']$ and investigate more fuzzy bioperator-approaches to properties of fuzzy topological spaces. The notions of new fuzzy bioperation-Separations are also introduced in this study. Furthermore, an attempt has been made to define fuzzy bioperation-continuous functions. Finally we obtained some relation of fuzzy bioperation-continuous functions and fuzzy bioperation-separation axioms.