CHAPTER-4

Functions with fuzzy γ-closed graphs and fuzzy γ-separations axioms.

4.1. Introduction:

Literature servey has revealed that with the help of a certain operation γ on topological space (X,T), Kasahara introduced the concept of γ-closed graph and Jankovic investigated some properties of functions with γ-closed graphs. Furthermore, Ogata studied some new separation axioms γ-T_i, $i = 1, 1/2, 2$. In this chapter, we define and study the above concepts with the help of q-coincidence in a fuzzy setting.

In the section 2, we have defined fuzzy γ-closed graphs, fuzzy γ-subcontinuity and then established their various properties.

In section 3, we have introduce and studied the concepts of fuzzy locally γ-closed function and particularly, fuzzy locally closed, fuzzy locally θ-closed and fuzzy locally δ-closed function. Then we have developed the notions of fuzzy γ-closed (fuzzy almost γ-closed) functions and generalized the concepts of fuzzy closed (almost-closed), fuzzy θ-closed(fuzzy almost θ-closed) and fuzzy δ-closed (fuzzy almost δ-closed) function. Attempts are also made to obtain some properties of said types of functions with fuzzy γ-closed graphs, fuzzy γ-continuity and fuzzy γ-compactness. Furthermore, using the fuzzy γ-open sets, some new separation axioms namely fuzzy γ-T_i spaces and some topological properties on them are presented in last section of this chapter.

4.2. Fuzzy γ-closed graphs and its properties

Definition 4.2.1: Let (X,T) and (Y,T') be two fts and γ an operation on T'. The graph $G(f)$ of function $f: X \to Y$ is said to be γ-closed iff for each fuzzy point $(P_x^\lambda, P_y^r) \in X \times Y - G(f)$, there exists open Q-nbds U and V of P_x^λ and P_y^r respectively such that $U \times \gamma(V) \cap G(f) = \varnothing$

Examples 4.2.2 :

(i) If γ is identity operation, then fuzzy γ-closedness of a graph is identical with the closedness of the graph.

(ii) γ is closure operation, then the fuzzy γ-closed graph G(f) is called fuzzy strong-closed graphs.

Lemma 4.2.3: A graph G(f) of a function $f: X \to Y$ is fuzzy γ-closed in $X \times Y$ if and only for each $(P_x^\lambda, P_y^r) \in X \times Y - G(f)$, there exists open Q-nbds U and V of P_x^λ and P_y^r such that $\gamma(V)\overline{q}f(U)$

Theorem 4.2.4: Let f be mapping from fuzzy topological space (X,T) into another fts (Y,T') and γ an operation on T'. Then for the following statements (1) – (3), the implications (1)$\Rightarrow$(2) and (2)$\Rightarrow$(3) hold. Furthr if γ is regular then (1) ,(2) ,(3) are equivalent to each other.

(1) f has fuzzy γ-closed graph

(2) If there exists a fuzzy filter base Φ in X converging to $P_x^\lambda \in S(X)$ such that $f(\Phi)$ fuzzy γ-accumulates to $p_y^r \in S(Y)$, then $f(p_x^\lambda) = p_y^r$.

(3) If there exists a filter base Φ in X converging to $P_x^\lambda \in S(X)$ such that $f(\Phi)$ fuzzy γ-converges to $p_y^r \in S(Y)$, then $f(P_x^\lambda) = P_y^r$.

***Proof*:** (1)$\Rightarrow$(2), If possible suppose $f(P_x^\lambda) \neq P_y^r$. Then $(P_x^\lambda, P_y^r) \in (X \times Y) - G(f)$. Since $G(f)$ is fuzzy γ-closed graph, there exists open Q-neighbourhood U and V of P_x^λ and P_y^r respectively such that $\gamma(V)\overline{q}f(U)$. Also $\Phi \to P_x^\lambda \in S(X)$, so $F \subseteq U$ for some $F \in \Phi$ and then we have $f(F) \subseteq f(U)$. Moreover, $f(\Phi)$ fuzzy γ-accumulates to $P_y^r \in S(Y)$, therefore we have $\gamma(V)qf(F)$. Thus $\gamma(V)qf(U)$, a contradiction.

(2) $\Rightarrow$ (3) Let there exists a filter base Φ in X converging to $P_x^\lambda \in S(X)$ such that $f(\Phi)$ fuzzy γ-converges to $P_y^r \in S(Y)$. Then $f(\Phi)$ fuzzy γ-accumulates to P_y^r. Thus, there exists a filter base Φ in X converging to P_x^λ such that $f(\Phi)$ fuzzy γ-accumulates to P_y^r. Therefore by assumption $f(P_x^\lambda) = P_y^r$.

Assume now that γ is regular. It suffices to show that (3) $\Rightarrow$ (1). Suppose that (1) does not hold. Then there exists a $(P_x^\lambda, P_y^r) \in (X \times Y) - G(f)$ such that $\gamma(V)qf(U)$ for every open Q-neighbourhood U and V of P_x^λ and P_y^r. Consider $\Phi = \{U \cap f^{-1}(\gamma(V))\}$. If A_1, $A_2 \in \Phi$ then for $U_1, U_2 \in N^Q(p_x^\lambda)$ and $V_1, V_2 \in N^Q(p_y^r)$, then

$A_1 \cap A_2 = (U_1 \cap U_2) \cap (f^{-1}(\gamma(V_1)) \cap f^{-1}(\gamma(V_2)) = U_3 \cap f^{-1}(\gamma(V_1) \cap \gamma(V_2))$. Since γ is regular, we have $A_1 \cap A_2 \supseteq U_3 \cap f^{-1}(\gamma(V_3)) = A_3$ (say). Thus Φ is fuzzy filterbase. Obiviously Φ converges to p_x^λ and $f(\Phi)$ γ-converges p_y^r. Then by (3) $f(p_x^\lambda) = p_y^r$, which is absurd.

Definition 4.2.5: Let (X,T), (Y,T') be topological spaces, and γ be an operation on T' A mapping $f: X \to Y$ is said to be fuzzy $\gamma -$subcontinuous if for every convergent filter base Φ in X, the filter base $f(\Phi)$ fuzzy γ-accumulates to some fuzzy point of Y.

Theorem 4.2.6: A function $f: X \to Y$ is fuzzy γ-continuous at P_x^λ iff for every filter base Φ in X converging to P_x^λ, the filter base $f(\Phi)$ fuzzy γ-converges to $f(P_x^\lambda)$.

***Proof*:** Let the function $f: X \to Y$ be fuzzy γ-continuous and Φ be any fuzzy filter base converging to $P_x^\lambda \in S(X)$. Then for each open Q-nbd V of $f(p_x^\lambda) \in S(Y)$, there exists an open Q-nbd U of P_x^λ such that $f(U) \subseteq \gamma(V)$. Since Φ is fizzy γ-converging to P_x^λ, there exists $F \in \Phi$ such that $F \subseteq U$. This implies $f(F) \subseteq f(U)$. Therefore $f(F) \subseteq \gamma(V)$. Also we have $P_x^\lambda \in Cl(A)$ for every $A \in \Phi$ and so $f(P_x^\lambda) \in f(Cl(A)) \subseteq Cl(f(A)) \subseteq Cl_\gamma(f(A))$. Thus $f(P_x^\lambda) \in Cl_\gamma(f(A))$. Hence $f(\Phi)$ fuzzy γ-converges to $f(P_x^\lambda)$. Conversely, let $P_x^\lambda \in S(X)$ and $V \in N^Q(f(P_x^\lambda))$.Since $\Phi = N^Q(P_x^\lambda) \to P_x^\lambda$, and $f(\Phi)$ fuzzy γ-converges to $f(P_x^\lambda)$ and so there exists $F \in \Phi$ such that $f(F) \subseteq \gamma(V)$. Hence f is fuzzy γ-continuous.

Theorem 4.2.7: Fuzzy γ-continuous mapping is fuzzy γ-subcontinuous.

***Proof*:** Let Φ be a fuzzy filterbase in X converging to $p_x^\lambda \in$ S(X). Then by theorem 4.2.6 fuzzy filterbase $f(\Phi)$ converges to $p_y^r \in$ S(Y). By theorem 3.5.4, $f(\Phi)$ γ-accumulates to p_x^r. Consequently f is fuzzy γ-subcontinuous

Theorem 4.2.8: Let (X,T), (Y,T') be topological spaces, and γ an operation on T'. If $f: X \to Y$ is fuzzy $\gamma-$subcontinuous with fuzzy γ-closed graph, then f is fuzzy γ-continuous.

***Proof*:** Suppose f is not fuzzy γ-continuous. Then there exists a $P_x^\lambda \in S(X)$ and filter base Φ in X converging to P_x^λ such that $f(\Phi)$ does not fuzzy γ-converge to $f(P_x^\lambda)$ and so there exists an open Q-neighbourhood V of $f(P_x^\lambda)$ such that $f(F) \subseteq (\gamma(V))^C$ which implies $f^{-1}(f(F) \subseteq f^{-1}((\gamma(V))^C)$. But we have $F \subseteq f^{-1}(f(F))$.

Therefore $F \subseteq f^{-1}((\gamma(V))^C)$. Let $\Psi' = \{F \bigcap f^{-1}((\gamma(V))^C) : F \in \Phi$. Then clearly Ψ' is filter base in X. Also Φ is contained in the filter Ψ generated by Ψ', and so Ψ converges to P_x^λ. Since f is fuzzy γ-subcontinuous, the filter base $f(\Psi)$ fuzzy γ-converges to some $P_y^r \in S(Y)$. Hence by theorem 4.2.4 (3), we have $f(P_x^\lambda) = P_y^r$, which is absurd. Hence f is fuzzy γ-continuous.

Theorem 4.2.9: If (Y,T') is fuzzy γ-compact space for some operation γ on T', then every mapping f from fts (X,T) into (Y,T') is fuzzy γ-subcontinuous.

Proof: Let Φ be a convergent fuzzy filter base in X. Then by theorem 3.5.11 the fuzzy filter base $f(\Phi)$ fuzzy γ-accumulates to some fuzzy point of Y. Thus f is fuzzy γ-subcontinuous.

Theorem 4.2.10: Let f be a mapping from fts (X,T) into fts (Y,T') and γ an operation on T'. If (Y,T') is fuzzy γ-compact and f has fuzzy γ-closed graph, then f is fuzzy γ-continuous.

Proof: By theorem 4.2.9 f is fuzzy γ-subcontinuous, and hence it is fuzzy γ-continuous by theorem 4.2.8

4.3. Some closed and open functions in fuzzy topological spaces.

Definition 4.3.1: Let (X,T), (Y,T') be fts and γ an operation on T'. A mapping $f:(X,T) \to (Y,T')$ is called fuzzy locally γ-closed if for each open Q-neighbourhood U of fuzzy point $p_x^\lambda \in$ S(X), there is a open Q-neighbourhood V of p_x^λ such that $V \subseteq U$ and $f(V)$ is fuzzy γ-closed in Y.

Example 4.3.2:

(1) If γ is identity operation then fuzzy locally γ-closed mapping is called fuzzy locally closed mapping

(2) If γ is closure operation then fuzzy locally γ-closed mapping is called fuzzy locally θ- closed mapping.

(3) If γ is interior-closure operation then fuzzy locally γ-closed mapping is called fuzzy locally δ- closed mapping.

Definition 4.3.3: Let γ be an operation on T'. A mapping $f:(X,T)\to(Y,T')$ is called

(1) fuzzy γ-closed if $f(A)$ is γ-closed set in Y for each fuzzy closed set A in X.

(2) fuzzy γ-open if $f(A)$ is γ-open set in Y for each fuzzy opn set A in X

Examples 4.3.4: (1) If γ is identity operation then fuzzy γ-closed (γ-open) mapping is coincides with fuzzy closed (fuzzy open) [21]

(2) If γ is closure operation then fuzzy γ-closed (γ-open) mapping is called fuzzy

θ-closed (fuzzy θ-open) [20]

(3) If γ is interior-closure operation then fuzzy γ-closed (γ-open) mapping is called fuzzy δ- closed (δ-open)

Definition 4.3.5: Let γ be an operation on T'. A mapping $f:(X,T)\to(Y,T')$ is called fuzzy almost γ-closed if $f(A)$ is γ-closed set in Y for each fuzzy regularly-closed set A in X

Example 4.3.6: (1) If γ is identity operation then fuzzy γ-closed mapping is coincides with fuzzy almost closed mapping

(2) If γ is closure operation then fuzzy almost γ-closed mapping is called fuzzy almost θ-closed mapping.

(3) If γ is interior-closure operation then fuzzy almost γ-closed mapping is called fuzzy almost δ- closed mapping.

Remark 4.3.7: If a function $f:(X,T)\to(Y,T')$ is fuzzy γ-closed, then it is fuzzy almost γ-closed function.

Proof: Let A be a fuzzy regular closed set X. Then A is fuzzy closed subset of X and hence by assumption $f(A)$ is fuzzy γ-closed set of Y. This shows that f is almost γ-closed function.

Remark 4.3.8: If a function $f:(X,T)\to(Y,T')$ is fuzzy almost γ-closed function where (X,T) is fuzzy regular space and γ is an operation on T', then it is fuzzy locally γ-closed function.

Lemma 4.3.9: Let (X,T) and (Y,T') be fts and γ an operation on T'. If a function $f:(X,T)\to(Y,T')$ is fuzzy locally γ-closed and has closed point inverses, then f has a fuzzy γ-closed graph.

Proof: Let $(p_x^\lambda, p_y^r)\in X\times Y-G(f)$. Then $p_x^\lambda\notin f^{-1}(p_y^r)$ and since $f^{-1}(p_y^r)$ is fuzzy closed, there exists an open Q-neighbourhood U of p_x^λ such that $U\overline{q}f^{-1}(p_y^r)$. The fuzzy locally γ-closedness of f implies that there is an open Q-neighbourhood V of p_x^λ such that $V\subseteq U$ and $f(V)$ is fuzzy γ-closed in Y. Since $f(p_x^\lambda)qf(V)$ and $f(p_x^\lambda)\neq p_y^r$, then $p_y^r\overline{q}f(V)$. This means $p_y^r\notin f(V)$ .Then there exists an open Q-neighbourhood W of p_y^r such that $f(V)\overline{q}\gamma(W)$ and hence, by Lemma 4.2.3 it follows that f has a fuzzy γ-closed graphs.

Lemma 4.3.10: Let (X,T) and (Y,T') be fts and γ an operation on T'. If a function $f:(X,T)\to(Y,T')$ is fuzzy almost γ-closed with fuzzy θ- closed point inverses, then f has a fuzzy γ-closed graphs.

Proof: Let $(p_x^\lambda, p_y^r)\in X\times Y-G(f)$. Then $p_x^\lambda\notin f^{-1}(p_y^r)$ and since $f^{-1}(p_y^r)$ is fuzzy fuzzy θ-closed, there exists an open Q-neighbourhood U of p_x^λ such that

$Cl(U)\overline{q}f^{-1}(p_y^r)$. Since Cl(U) is fuzzy regularly-closed, the fuzzy almost γ-closedness of f implies that $f(Cl(U))$ is fuzzy γ-closed in Y. Since $f(p_x^\lambda)qf(Cl(U))$ and $f(p_x^\lambda) \neq p_y^r$, then $p_y^r\overline{q}f(cl(U))$. This means $p_y^r \notin f(Cl(U))$. Then there exists an open Q-neighbourhood W of p_y^r such that $f(Cl(U))\overline{q}\gamma(W)$. Since $U \subseteq Cl(U)$ implies $f(U) \subseteq f(Cl(U))$, it follows that $f(U)\overline{q}\gamma(W)$ and hence, by Lemma 4.2.3 it follows that f has a fuzzy γ-closed graph.

Theorem 4.3.11: Let (X,T) and (Y,T') be fts and γ be an operation on T'. If a function $f:(X,T) \to (Y,T')$ is fuzzy almost γ-closed with fuzzy closed point inverses and X is a fuzzy regular space, then f has a fuzzy γ-closed graphs.

Proof: Since fuzzy θ-closure and closure coincide for subsets of a fuzzy regular space, it follows from above Lemma 4.3.10.

Theorem 4.3.12: Let (X,T) and (Y,T') be fts and γ be an operation on T'. If a function $f:(X,T) \to (Y,T')$ is fuzzy almost γ-closed function with fuzzy θ- closed point inverses and (Y,T') is fuzzy γ-compact, then f is fuzzy γ-continuous.

***Proof*:** It follows from Lemma 4.3.10 and theorem 4.2.10.

Theorem 4.3.13: Let (X,T) and (Y,T') be fts and γ be an operation on T'. If a function $f:(X,T) \to (Y,T')$ is fuzzy locally γ-closed function with fuzzy closed point inverses and (Y,T') is fuzzy γ-compact, then f is fuzzy γ-continuous.

***Proof*:** It follows from Lemma 4.3.9 and theorem 4.2.10.

Theorem 4.3.14: If f is a fuzzy almost γ-closed function from a fuzzy regular space (X,T) into a fuzzy γ-compact space (Y,T') such that $f^{-1}(p_y^r)$ is fuzzy closed for every $p_y^r \in S(Y)$ then f is fuzzy γ-continuous .

***Proof*:** Since fuzzy θ-closure and closure coincide for subsets of a fuzzy regular space, it follows from Lemma 4.3.10 that f has a fuzzy γ-closed graph. Since (Y,T') is fuzzy γ-compact, then by theorem 4.2.10 f is fuzzy γ-continuous.

Theorem 4.3.15: Let (X,T) and (Y,T') be fts and γ an operation on T'. If a function $f:(X,T)\to(Y,T')$ is fuzzy almost open function with fuzzy closed graph, then f has a fuzzy strongly-closed graph.

***Proof*:** Let $(p_x^\lambda, p_y^r)\in X\times Y - G(f)$. Since f has a fuzzy closed graphs, there exists an open Q-neighbourhoods U and V of p_x^λ and p_y^r such that $f(U)\overline{q}V$. This implies $U\overline{q}f^{-1}(V)$ and $U\overline{q}Cl(f^{-1}(V))$. Since f is fuzzy almost open , $U\overline{q}f^{-1}(Cl(V))$. Hence $f(U)\overline{q}Cl(V)$. So f has a fuzzy strongly-closed graph.

Theorem 4.3.16: Let (X,T) and (Y,T') be fts and γ be an operation on T'. If a function $f:(X,T)\to(Y,T')$ is fuzzy almost-open, almost-closed function with fuzzy θ-closed point inverses, then f has a fuzzy strongly-closed graphs.

***Proof*:** It follows from Lemma 4.3.10 (when γ is identity operation) and theorem 4.3.15

Lemma 4.3.17: If a function $f:(X,T)\to(Y,T')$ is fuzzy almost γ-closed injection function where (X,T) is fuzzy Hausdorff and γ is an operation on T', then f has a fuzzy γ-closed graph.

***Proof*:** Since (X,T) is fuzzy Hausdorff, its fuzzy points are fuzzy θ-closed and hence f has fuzzy θ-closed point inverse. Now by lemma 4.3.10, it follows that f has a fuzzy γ-closed graph.

Theorem 4.3.18: Let f be a fuzzy almost γ-closed injection function from a fuzzy Hausdorf space (X,T) into a fuzzy γ-compact space (Y,T') and γ be an operation on T', then f is fuzzy γ-continuous.

***Proof*:** It follows from the Lemma 4.3.17 and Theorem 4.2.10.

Lemma 4.3.19: If a function $f:(X,T)\rightarrow(Y,T')$ is fuzzy almost- open function with closed graph, then f has a fuzzy strongly-closed graph.

***Proof*:** Let $(p_x^{\lambda}, p_y^{r})\in X\times Y-G(f)$. Since f has a fuzzy closed graph, there exists an open Q-neighbourhoods U and V of p_x^{λ} and p_y^{r} such that $f(U)\overline{q}V$. This implies that $U\overline{q}f^{-1}(V)$. Therefore $U\overline{q}Cl(f^{-1}(V))$. Since f is fuzzy almost open function, $U\overline{q}f^{-1}(Cl(V))$. Hence $f(U)\overline{q}Cl(V)$. So f is fuzzy strongly-closed graph

4.4. Fuzzy γ-separation Axioms:

Definition 4.3.1: A fts (X,T) is called:

(1) Fuzzy γ-T_1 iff for any $p_x^{\lambda}, p_y^{k}\in$ S(X) and $p_x^{\lambda}\neq p_y^{k}$, there exists open Q-neighbourhoods U and V of p_x^{λ} and p_y^{k} respectively such that $p_y^{k}\overline{q}\gamma(U)$ and $p_x^{\lambda}\overline{q}\gamma(V)$

(2) Fuzzy γ-T_2 iff for any $p_x^{\lambda}, p_y^{k}\in$ S(X) and $p_x^{\lambda}\neq p_y^{k}$, there exists open Q-neighbourhoods U and V of p_x^{λ} and p_y^{k} respectively such that $\gamma(U)\overline{q}\gamma(V)$.

Theorem 4.3.2: If a space (X,T) is fuzzy γ-T_2, then it is fuzzy γ-T_1.

***Proof*:** Let (X,T) be a fuzzy γ-T_2 space. Let $p_x^{\lambda}, p_y^{k}\in$ S(X) and $p_x^{\lambda}\neq p_y^{k}$,then there exists open Q-neighbourhoods U and V of p_x^{λ} and p_y^{k} respectively such that

$\gamma(U)\overline{q}\gamma(V)$. Since $p_x^\lambda q\gamma(U)$ and $p_y^k q\gamma(V)$,therefore $p_x^\lambda\overline{q}\gamma(V)$ and $p_y^k\overline{q}\gamma(U)$. Hence (X,T) is γ-T_1.

Theorem 4.3.3: A space (X,T) is fuzzy γ-T_1 if and only any fuzzy singleton in X is a fuzzy γ-closed set.

***Proof*:** (Necessity): Let (X,T) be a fuzzy γ-T_1 and $p_x^\lambda \in$ S(X). Since $p_x^\lambda \subseteq cl_\gamma(p_x^\lambda)$, so it is only need to prove $cl_\gamma(p_x^\lambda) \subseteq p_x^\lambda$ Let $p_y^k \notin p_x^\lambda$.Then $p_x^\lambda \neq p_y^k$ and by assumption, there exists an open Q-neighbourhood V of p_y^k respectively such that $p_y^k\overline{q}\gamma(U)$. This implies $p_y^k \notin cl_\gamma(p_x^\lambda)$. Thus $cl_\gamma(p_x^\lambda) \subseteq p_x^\lambda$ and hence $cl_\gamma(p_x^\lambda) = p_x^\lambda$. This shows that p_x^λ is γ-closed set.

(sufficiency): Let $p_x^\lambda, p_y^k \in$ S(X) and $p_x^\lambda \neq p_y^k$. Since p_x^λ and p_y^k are both γ-closed set, $cl_\gamma(p_x^\lambda) = p_x^\lambda$ and $cl_\gamma(p_y^k) = p_y^k$. Since $p_x^\lambda \neq p_y^k$, then $p_y^k \notin cl_\gamma(p_x^\lambda)$ and $p_x^\lambda \notin cl_\gamma(p_y^k)$. Therefore, there exists open Q-neighbourhoods U and V of p_x^λ and p_y^k respectively such that $p_x^\lambda\overline{q}\gamma(V)$ and $p_y^k\overline{q}\gamma(U)$. This implies (X,T) is fuzzy γ-T_1 space.

Theorem 4.3.4: Suppose $\gamma: T \to I^X$ is regular operation. If (X,T_γ) is a fuzzy T_2 space then (X,T) is a fuzzy γ-T_2.

Proof: $p_x^\lambda, p_y^k \in$ S(X) and $p_x^\lambda \neq p_y^k$. Since (X,T_γ) is a fuzzy T_2 space, then there exists open Q-neighbourhoods U, V ($\in$ $T_\gamma \subseteq T$) of p_x^λ and p_y^k respectively such that $\gamma(U)\overline{q}\gamma(V)$. Thus (X,T) is a fuzzy γ-T_2.

Definition 4.3.5: Let (X,T) be a fts and γ an operation on T. A fuzzy set $A \in I^X$ is called γ-generalized closed (γ-g-closed, for short) if $cl_\gamma(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy γ-open in (X,T).

Theorem 4.3.6: Every fuzzy γ-closed set is fuzzy γ-g-closed.

***Proof*:** Obvious. The converse is not true as shown by the following example.

Example 4.3.7: Let X = {x,y} and $T = \{X, \varnothing, p_y^{0.7}\}$. Define $\gamma : T \to I^X$ by $\gamma(U) = cl(U)$ for each $U \in T$.

Let $A = p_x^{0.5} \cup p_y^{0.6}$. Then A is fuzzy γ-g-closed set but not fuzzy γ-closed set.

Definition 4.3.8: A space (X,T) is called a fuzzy γ-$T_{1/2}$ space if every fuzzy γ-g.closed set of (X,T) is fuzzy γ-closed.

We conclude this chapter with following teorem on $\gamma - T_{1/2}$ space

Theorem 4.3.9: For each $p_x^\lambda \in$ S(X), p_x^λ is γ-closed or $(p_x^\lambda)^C$ is fuzzy γ-g.closed set in (X,T).

***Proof*:** Suppose p_x^λ is not γ-closed. Then $(p_x^\lambda)^C$ is fuzzy γ-open. Let U be any fuzzy γ-open set such that $(p_x^\lambda)^C \subseteq U$. Since $U = X$ is the only fuzzy γ-open, $cl_\gamma((p_x^\lambda)^C) \subseteq U$ Therefore $(p_x^\lambda)^C$ is fuzzy γ-g.closed set.