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3 (Sem–3/CBCS) PHY HC 1

2021

(Held in 2022)

PHYSICS

(Honours)

Paper : PHY–HC–3016

(Mathematical Physics–II)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : (each question carries **one** mark) 1×7=7

(a) Show that $P_n(-x) = (-1)^n P_n(x)$.

(b) $L_1(x) - L_0(x) = ?$

(c) Express the one-dimensional heat flow equation.

(d) $\int_0^\infty e^{-x} x^{2n-1} dx = ?$

(e) $\beta\left(\frac{1}{2}, \frac{1}{2}\right) = ?$

(f) Square matrix = Symmetric matrix + ?

(g) If, $\mu^{-1} M \mu = M'$, then show that $Tr\, M = Tr\, M'$.

2. Answer the following questions : (each question carries **2** marks) 2×4=8

(a) Show that $x = 0$ is a regular singular print for the following differential equation :

$$2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + \left(x^2 - 4\right) y = 0$$

(b) Can we express the one-dimensional Schrödinger's equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial t^2}+V\psi(x,t)=i\hbar\frac{\partial\psi}{\partial t}(x,t)$$

in terms of space dependent and time independent equations if V is a function of both x and t? Explain.

(c) Show that $\beta(l,m)=\beta(m,l)$.

(d) Show that the matrix

$$\sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

is Hermitian as well as unitary.

3. Answer ***any three*** questions from the following : (each question carries **5** marks) 5×3=15

(a) By the separation of variable method, solve the t-dependent part of the following equation : 5

$$\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$$

(b) If $\begin{pmatrix} x \\ y \end{pmatrix}$ transforms to $\begin{pmatrix} x' \\ y' \end{pmatrix}$ in the way —

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$ then

show that $x'^2 + y'^2 = x^2 + y^2$.

Verify that the transformation matrix is orthogonal. 2+3=5

(c) How many real numbers are required to express a general complex matrix of dimension 2×2? Show that a 2×2 Hermitian matrix of dimension 2×2 carries four real numbers. Also, show that a skew-Hermitian matrix of dimension 2×2 carries only the real numbers. 1+2+2=5

(d) Find the Fourier's series representing $f(x) = x,\ 0 < x < 2\pi$, and sketch its graph from $x = -4\pi$ to $x = +4\pi$.

3+2=5

(e) Show that

$$L'_n(x) - n\,L'_{n-1}(x) + n\,L_{n-1}(x) = 0 \,. \qquad 5$$

4. If, $y = \sum_{k=0}^{\infty} a_k\, x^{m+k}$ happens to be the power series solution of the equation,

$2x(1-x)\dfrac{d^2y}{dx^2} + (1-x)\dfrac{dy}{dx} + 3y = 0$, then show that $a_{k+1} = \dfrac{-2m-2k+3}{2m+2k+1}$ 10

Or

Show the following : 4+3+3=10

(1) $(n+1)\,P_{n+1} = (2n+1)\,x\,P_n - n\,P_{n-1}$

(2) $n\,P_n = x\,P'_n - P'_{n-1}$

(3) $P'_{n+1} - P'_{n-1} = (2n+1)\,P_n$

5. Solve the equation

$$\frac{\partial^2 \psi}{\partial x \partial t} = e^{-t} \cos x$$

given that, $\psi(t=0) = 0$ and $\left.\frac{\partial \psi}{\partial t}\right|_{x=0} = 0$

10

Or

Consider a vibrating string of length l fixed at both ends, given that

$y(0, t) = 0, \;\; y(l, t) = 0$

$y(x, 0) = f(x), \; \frac{\partial y}{\partial t}(x, 0) = 0; \;\; 0 < x < l$

Solve completely the equation of vibrating string

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$

10

6. If $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$, obtain A^{-1}.

From the matrix equation

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix},$$

obtain, a, b, c, d. 4+6=10

Or

Obtain the eigenvalues and eigenvectors of the matrix

$$M = \begin{pmatrix} 2 & -i \\ i & 2 \end{pmatrix}$$

and hence diagonalize the same. 4+6=10