2019

PHYSICS

(Major)

Paper : 5.3

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Quantum Mechanics)

(*Marks* : 40)

1. Answer any *four* questions as directed : 1×4=4

(a) Which one of the following does not support quantum nature of radiation?

(i) Photoelectric effect

(ii) Compton effect

(iii) Interference

(iv) Black-body radiation

(Select the correct one)

(b) The quantum mechanical form of the total energy operator of a particle moving in x-direction is

(i) $\hat{H} = \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + v(x)$

(ii) $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + v(x)$

(iii) $\hat{H} = \frac{\hbar}{2m}\frac{\partial}{\partial x} + v(x)$

(iv) $\hat{H} = -\frac{\hbar}{2m}\frac{\partial}{\partial x} + v(x)$

(Select the correct one)

(c) State complementary principle of Neil's Bohr.

(d) Show that the de Broglie wavelength of a particle of rest mass m_0 and kinetic energy k is given by

$$\lambda = \frac{hc}{\sqrt{k(k+2m_0c^2)}}$$

where h = Planck's constant.

(e) Select the correct statement :

(i) Schrödinger equation can be derived from Newton's laws of motion.

(ii) Schrödinger equation can be derived from Maxwell's equations of electromagnetic theory

(iii) Schrödinger equation can be derived from both Newton's laws of motion and Maxwell's equations of electromagnetic theory.

(iv) Schrödinger equation can not be derived from any known principle. It is a fundamental equation of quantum mechanics.

2. Answer any *three* questions : 2×3=6

(a) Prove that $[x, p_x] = i\hbar$.

(b) The threshold frequency for photoelectric emission in copper is $1 \cdot 1 \times 10^{15}$ Hz. Find the maximum kinetic energy of the photoelectron when light of frequency $1 \cdot 5 \times 10^{15}$ Hz is incident on a copper surface.

(c) Find the phase velocity and group velocity of the de Broglie waves of an electron whose kinetic energy is 500 eV.

(d) Normalise the wave function

$$\psi(x) = A \sin \frac{n\pi x}{L}, \quad 0 < x < L$$

where $n = 1, 2, 3, \ldots$.

3. Answer any *two* questions : 5×2=10

(a) *(i)* Using uncertainty relation, show that an electron cannot reside inside a nucleus.

(ii) An excited atom gives up its excess energy by emitting a photon of characteristic frequency. The average period that elapses between the excitation of an atom and the time it radiates is $1{\cdot}0\times10^{-8}$ second. Find the inherent uncertainty in the frequency of the photon. 3+2=5

(b) An incident X-ray photon of frequency ν_0 is scattered by a free electron at rest through an angle θ. Using relativistic expression of electron energy, show that the change in the wavelength of the photon is given by

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

where m_0 = rest mass of electron, h is the Planck's constant and c is the velocity of light. What is Compton wavelength? 4+1=5

(c) Briefly discuss the Davisson-Germer experiment and it implications. 5

4. Answer any *two* questions : 5×2=10

(a) *(i)* Mention the properties of well-behaved wavefunctions.

(ii) A particle limited to the x-axis has the wavefunction

$$\psi(x) = ax, \quad 0 < x < 1$$
$$= 0, \quad x < 0, x > 1$$

Find the probability that the particle can be found between $x = 0 \cdot 45$ and $x = 0 \cdot 55$. Also, find the expectation value $\langle x \rangle$ of the particle's position. (Here a is a constant.) 2+3=5

(b) Write one-dimensional time-dependent Schrödinger equation for a particle influenced by an arbitrary potential $V(x, t)$. Mention one important property of this equation. Starting from this equation, obtain one-dimensional time-independent Schrödinger equation. What is stationary state? 1+1+2+1=5

(c) Derive the continuity equation from the time-dependent Schrödinger equation of a particle moving in a real potential. Give the physical interpretation of the continuity equation you derived. 4+1=5

5. Answer any *two* questions : 5×2=10

(a) A particle of mass m is confined in a one-dimensional infinite potential well

$$V(x) = 0, \quad -a < x < a$$
$$= \infty, \quad |x| > a$$

Find the energy of the particle in nth quantum state.

(b) For a linear harmonic oscillator potential $V(x) = \frac{1}{2}kx^2$, show that the Schrödinger equation (time-independent) takes the form

$$\frac{d^2\psi(q)}{dq^2} + (\lambda - q^2)\psi(q) = 0$$

where

$$\lambda = \frac{2E}{\hbar\omega}, \; w = \sqrt{\frac{k}{\omega}}, \; q = \alpha x, \; \alpha = \sqrt{\frac{m\omega}{\hbar}}$$

Write the energy of the oscillator in nth quantum state. What is zero-point energy? 3+1+1=5

(c) One-dimensional potential barrier is defined by

$$V(x) = 0, \quad x < 0$$
$$= V_0, \quad 0 < x < a$$
$$= 0, \quad x > a$$

Discuss the motion of a particle when $E < V_0$, where E is the energy of the

particle. What is quantum mechanical tunnel effect? Give a direct experimental evidence of quantum mechanical tunnelling phenomenon. 3+1+1=5

GROUP—B

(Astrophysics)

(*Marks* : 20)

6. Answer any *three* of the following : 2×3=6

(a) Define the right ascension and declination of a celestial object.

(b) Define local sidereal time. A star has right ascension $\alpha = 6\text{ h }51\text{ m}$ and another star has $\alpha = 10\text{ h }52\text{ m}$. Which one of them rises earlier?

(c) The apparent magnitude of the sun is $-26 \cdot 7$ and that of Antares is +1. How much brighter does the sun appear than Antares?

(d) What do you mean by colour index? The colour indices, B-V of four stars are *(i)* $-0 \cdot 5$, *(ii)* 0, *(iii)* $0 \cdot 5$ and *(iv)* $1 \cdot 0$. Which one of them is the hottest?

(e) What is a parsec? Relate parsec to light year.

7. Answer any *two* of the following : 4×2=8

(a) Draw a neat HR diagram showing the main sequence, the sun, red giant and red supergiant. Explain why the life of a massive star is shorter. 2+2=4

(b) What is the energy-generation mechanism inside the main sequence stars? Which reaction cycle dominates in the sun like stars? Discuss carbon-nitrogen-oxygen (CNO) cycle. 1+1+2=4

(c) Define the absolute magnitude of a star. Obtain the relation connecting absolute magnitude with apparent magnitude and distance of the star in parsec. 1+3=4

8. Write short notes on any *two* of the following : 3×2=6

(a) Trigonometric parallax

(b) White dwarf

(c) Spectral classification of stars

(d) Celestial coordinates

The following data can be used when required :

$e = 1 \cdot 6 \times 10^{-19} c$, $c = 3 \times 10^{8}$ m/s

$h = 6 \cdot 6 \times 10^{-34}$ Js, $m_e = 9 \cdot 1 \times 10^{-31}$ kg

1 AV = $1 \cdot 49 \times 10^{8}$ km

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