2019

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 30)

1. Answer the following questions : 1×4=4

(a) Define a simply connected region in complex plane.

(b) Find out the conjugate of a complex number $7+6i$.

(c) Define a singular point of a function.

(d) Give the Euler's formula.

2. (a) State De Moivre's theorem. 2

(b) Find the modulus and argument of the complex number

$$\frac{1+2i}{1-(1-i)^2}$$ 2

3. (a) Examine whether the function $f(z) = e^z$ is an analytic function or not. 3

(b) Demonstrate a graphical representation of complex variable through Argand diagram. 2

4. State and prove Cauchy's integral theorem. 5

5. Using Cauchy's integral formula, evaluate

$$\int_C \frac{z}{(z^2 - 3z + 2)}\, dz$$

where C is the circle $|z-2| = \frac{1}{2}$. 5

6. *(a)* *(i)* Develop the Taylor series expansion and find the radius of convergent for $\ln z$ about $z_0 = 1$. 5

(ii) Evaluate $\oint_C \frac{dz}{z}$, where C is a circle of unit radius. 2

Or

(b) *(i)* Show that

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \pi$$ 5

(ii) Give the Laurent series expansion for $f(z)$. 2

GROUP—B

(Classical Mechanics)

(*Marks* : 30)

7. Answer the following questions/Choose the correct option : 1×4=4

(a) What is cyclic or ignorable coordinate?

(b) What is a central force?

(c) A particle is constrained to move along the inner surface of a fixed hemispherical bowl. The number of degrees of freedom of the particle is

(i) one

(ii) two

(iii) three

(iv) six

(d) For a conservative system, the potential energy does not depend upon

(i) force

(ii) generalised coordinate

(iii) generalised velocity

(iv) All of the above

8. Answer any *two* of the following questions : $2\times2=4$

(a) What do you understand by holonomic and non-holonomic constraints?

(b) Explain reduced mass in the context of two-body central force problem.

(c) What are generalised coordinates?

9. Answer any *two* of the following questions : $3\times2=6$

(a) State Kepler's laws of planetary motion.

(b) Show that Hamiltonian H is a constant of motion if the Lagrangian L is not an explicit function of time.

(c) Show that a two-body central force problem can be reduced to one-body problem.

10. *(a)* The Lagrangian of a problem is

$$L = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2) + V(r)$$

Identify the cyclic coordinate and the corresponding conservation law for the problem. 4

Or

Show that for a particle moving under a central force, the total mechanical energy of the particle is conserved.

(b) Use Lagrange's equations to find the equation of motion of a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis. 5

Or

Establish the Hamiltonian and equations of motion of a simple pendulum.

11. Derive Lagrange's equation of motion from Hamilton's principle for a conservative system. 7

Or

Derive Lagrange's equation of motion for a conservative system using D'Alembert's principle.

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