2016

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following : 1×10=10

(a) What does the singular solution of a differential equation represent?

(b) Give one example of a linear differential equation.

(c) Write the complementary function of

$$(D^2+4)y = x^2 \sin 2x$$

(d) Write the form of a total differential equation.

(e) What does the complete integral of a first-order partial differential equation represent?

(f) Give an example of a first-order and second-degree differential equation.

(g) When a total differential equation is said to be exact?

(h) Define linear partial differential equation.

(i) What is an ordinary differential equation?

(j) Write down the order and degree of

$$x^2\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^4 = 0$$

2. Answer any *five* of the following questions : 2×5=10

(a) Prove that $y = \sin x$ is a part of complementary function of

$$(\sin x - x\cos y)y'' - x\sin x y' + y\sin x = 0$$

(b) Give a geometrical interpretation of the equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, where P, Q, R are the functions of x, y, z.

(c) Find the differential equation of the family of curves $y = me^{2x} + ne^{-2x}$ for different values of m and n.

(d) Solve $\frac{dy}{dx} = \sec(x+y)$.

(e) Find a partial differential equation by eliminating a and b from $az + b = a^2x + y$.

(f) Solve $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$.

3. Answer any *five* of the following questions : 4×5=20

(a) Reduce $y = 2px + y^2 p^3$ to Clairaut's form using the transformation $y^2 = v$ and hence solve it.

(b) Solve $(D^2 - 2D + 1)y = \cos 3x$.

(c) Solve $(ax + hy + g)dx + (hx + by + f)dy = 0$.

(d) Solve $\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$.

(e) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + a^2 y = \sec ax$.

(f) Solve :

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$$
$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4\cos t - 3\sin t$$

4. Answer any *five* of the following questions : 6×5=30

(a) Solve $\frac{d^4y}{dx^4} + m^4 y = 0$.

(b) Verify the condition of integrability for

$$(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$$

and solve it.

(c) Solve $z(z^2 + xy)(px - qy) = x^4$.

(d) Find a complete integral of

$$z^2(p^2z^2+q^2)=0$$

by Charpit's method.

(e) Form a partial differential equation by eliminating the arbitrary function ϕ from

$$\phi(x^2+y^2+z^2,\ z^2-2xy)=0$$

(f) Find $f(y)$ s.t. $\{(yz+z)/x\}\,dx - z\,dy + f(y)\,dz = 0$ is integrable. Also find the corresponding integral.

5. Answer *either* (a) and (b) *or* (c) and (d) : 5+5=10

(a) Solve $x^2y''+xy'-y=0$, given that $x+\frac{1}{x}$ is one integral.

(b) If u and v are two independent particular integrals of the equation

$$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)\,y=0$$

prove that $u\frac{dv}{dx}-v\frac{du}{dx}=c\cdot e^{-\int P\,dx}$.

(c) Solve $y''\cos x + y'\sin x - 2y\cos^3 x = 2\cos^5 x$ by changing the independent variable.

(d) Find the equation of the integral surface of the differential equation

$$(x^2-yz)\,p+(y^2-zx)\,q=z^2-xy$$

which passes through the lines $x=1$, $y=0$.

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